Lecture 6 - Electroweak Phenomenology

We will begin our exploration of electroweak (EW) pheno by studying how the neutral currents can be used to verify the couplings and to measure $\sin^2\theta_W$.

We will study three scattering processes at low energies $E \ll m_W$ so that Fermi theory is a good approximation. We will assume we have a beam of neutrinos (either from a reactor (ν_e) or from μ decays (ν_μ)) that scatter off electrons in some fixed target (effectively at rest):

1) $\nu_\mu e^- \to \nu_\mu e^-$
2) $\bar{\nu}_\mu e^- \to \bar{\nu}_\mu e^-$
3) $\bar{\nu}_e e^- \to \bar{\nu}_e e^-$

Note that it was the discovery of neutral currents that prompted the need for the Z^0 boson.

Recall that Fermi theory is given by

$$\mathcal{L} \supset \underbrace{-\frac{G_F}{\sqrt{2}} J_\mu^+ J^{-,\mu}}_{\text{charged current}} - \frac{4G_F}{\sqrt{2}} \underbrace{J_\mu^0 J^{0,\mu}}_{\text{neutral current}}$$

Note: Charged current mediates decays (e.g. muon decay) while the neutral current does not since it is "flavor diagonal."

$$J^{\mu,0} = \sum_i \bar{\psi}_i \left(g_V(i) - g_A(i)\gamma^5\right)\psi_i$$

$\Rightarrow$ test via scattering.

SM predicts
$$g_V = \frac{1}{4}\left[2T^3 - 4s_W^2 Q\right]$$
$$g_A = \frac{1}{4}\left[2T^3\right]$$

Want to determine consistency with this prediction and measurement of s_W^2.

Let us introduce a generalized neutral current:

$$\tilde{J}_\mu^0 = \sum_{l=e,\mu}' \bar{l}\gamma^\mu\left(g_V^l - g_A^l\gamma^5\right)l + \sum_{\nu=\nu_e,\nu_\mu}' \bar{\nu}\gamma^\mu\left(\frac{1}{4} - \frac{1}{4}\gamma^5\right)\nu$$

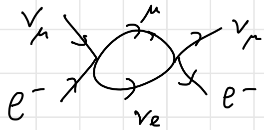
Same as SM for simplicity (s_W^2 independent)

Goal is to test consistency with

$$g_V^\ell = \frac{1}{4}\left[-1 + 4 s_w^2\right] \;;\; g_A^\ell = -\frac{1}{4}$$

Note that tree-level (1) + (2) only get contributions from neutral current.

(There is a 1-loop charged current contribution



+ ...)

Tree-level (3) has both neutral and charged current contributions.

* You will compute (1), (2), + (3) at tree-level in H/W so we will just quote the results here.

$$\sigma_1 = \sigma(\nu_\mu e^- \to \nu_\mu e^-) = \frac{G_F^2 s}{\pi}\left[(g_V^e + g_A^e)^2 + \frac{1}{3}(g_V^e - g_A^e)^2\right]$$

w/ $s = E_{cm}^2 = 2 m_e E_\nu$ (fixed target experiment)

First, let's estimate the size of σ_1.

Assume $g_V \sim g_A \sim 1$; $G_F \sim (100\,\text{GeV})^{-2}$; $E_{cm} \sim \text{GeV}$

($E_{cm} \ll G_F$ so that EFT is valid approx)

$$\sigma_1 \sim \frac{G_F^2 E_{cm}^2}{\pi} \sim 4 \times 10^{-11} \left(\frac{E_{cm}}{1\,\text{GeV}}\right)^2 \text{GeV}^{-2}$$

By comparison, QED cross sections are

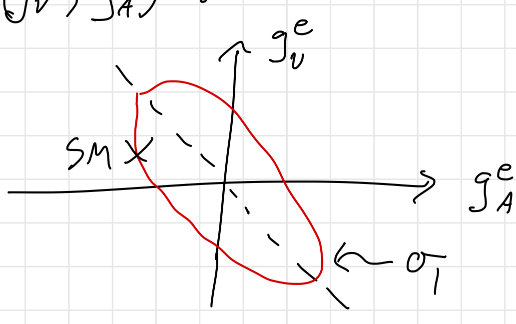
$$\sigma_{QED} \sim \sigma(e^+e^- \to \mu^+\mu^-) = \frac{\pi \alpha_{em}^2}{8 E_{cm}^2} \sim 6\times 10^{-5}\left(\frac{1\,\text{GeV}}{E_{cm}}\right)^2 \text{GeV}^{-2}$$

$$\Rightarrow \frac{\sigma_W}{\sigma_{QED}} \sim 8\times 10^{-7}\left(\frac{E_{cm}}{1\,\text{GeV}}\right)^4$$

$\Rightarrow$ Weak force is "weak" at low energies

At high energies $E \gg m_W$, need UV completion w/ $W + Z$ and then weak force is stronger than QED. Eg $\frac{\sigma_W}{\sigma_{QED}} \sim 1$ at $E_{cm} = 30$ GeV

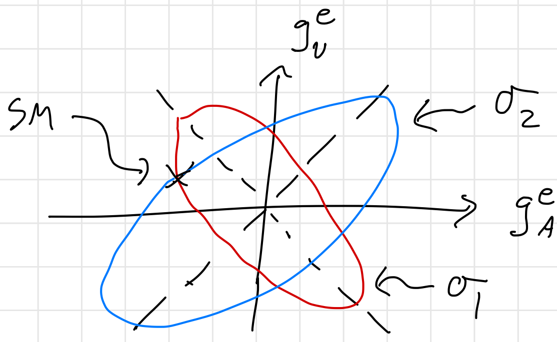
- Measurement of σ_1 gives family of consistent (g_V, g_A):



To break degeneracy, let's turn to process 2:

$$\sigma_2 = \sigma(\bar{\nu}_\mu e^- \to \bar{\nu}_\mu e^-) = \frac{G_F^2 s}{\pi}\left[(g_V^e - g_A^e)^2 + \frac{1}{3}(g_V^e + g_A^e)^2\right]$$

Note $\sigma_2 = \sigma_1\big|_{g_A^e \to -g_A^e}$ $\Rightarrow$ reflected ellipse



Note radius is bigger because of different g dependence

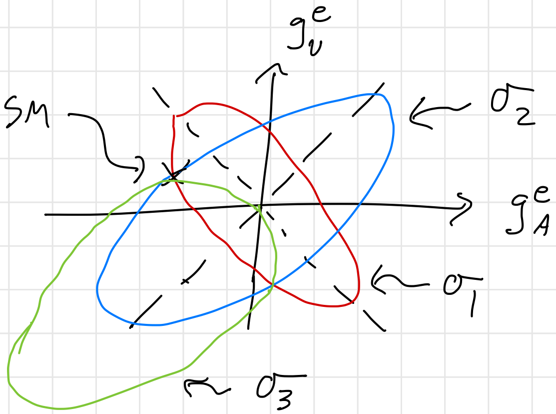
Now we have only a discrete ambiguity, the four places where the ellipses cross, charictorized by 2 Z_2's : $\begin{cases} g_V \to -g_V \\ g_A \to -g_A \end{cases}$ and $\begin{cases} g_V \to g_A \\ g_A \to g_V \end{cases}$

First degeneracy because only have neutral current contribution $\Rightarrow$ need interference

$\Rightarrow$ want process with charged current contribution.

This motivates measuring (3)

$$\sigma_3 = \sigma(\bar{\nu}_e e^- \to \bar{\nu}_e e^-) = \frac{G_F^2 s}{\pi}\left[(g_V^e - g_A^e)^2 + \left(\overset{\text{charged current}}{1} + g_V^e + g_A^e\right)^2\right]$$



Now we only have 2 solutions. We can understand the origin of this degeneracy as coming from $\left(P_{L,R} = \frac{1 \mp \gamma^5}{2}\right)$

$$g_V - g_A \gamma^5 = \underbrace{(g_V + g_A)}_{g_L} P_L + \underbrace{(g_V - g_A)}_{g_R} P_R$$

So the ambiguity is $g_R \to -g_R$ and $g_L \to g_L$, we need an observable that is sensitive to the sign of g_R. We can understand this sensivity because the particles are taken to be massless $\Rightarrow$ helicity is conserved so that $\sigma = \sum_h \sigma_h$, i.e., these processes do not interfere.

Since g_L determines left handed processes while g_R determines right handed processes, there are no $g_L g_R$ terms. Also, the charged current only couples to left handed particles, so can only get an interference term with g_L (linear term) but no linear g_R term.

We have 2 options. The most obvious is to include finite mass effects. But this is hopeless for our processes due to tiny neutrino mass. The second option is to use a process with a QED contribution. Since QED is "vector like" (couples to L and R equally), it can interfere with the right handed contribution to the weak interactions. The solution was to devise an observable that is sensitive to the weak interactions, the so-called "forward-backward asymmetry" for $e^+e^- \to \mu^+\mu^-$:

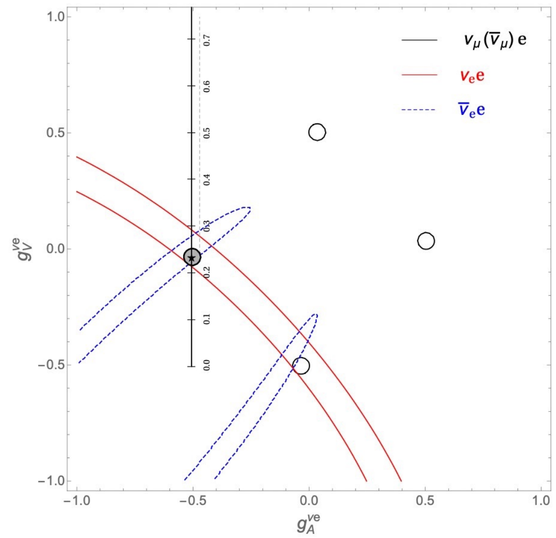
$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} \qquad \sigma_F = \sigma\left(\theta_\mu > \frac{\pi}{2}\right) \qquad \sigma_B = \sigma\left(\theta_\mu < \frac{\pi}{2}\right)$$

We will discuss this in more detail later, but for now we can quote the result

$$A_{FB} \simeq \frac{-3 G_F s}{\sqrt{2} e^2} (2 g_A^e)^2$$

$\Rightarrow$ measurement of magnitude of g_A^e which resolves the ambiguity.

All together, this provided us with a measurement of $\sin^2\theta_W \simeq 0.23$.



← See PDG review on EW Model and Constraints

Figure 10.1: Allowed contours in $g_A^{\nu e}$ *vs.* $g_V^{\nu e}$ from neutrino-electron scattering and the SM prediction as a function of $\hat{s}_Z^2$. (The SM best fit value, $\hat{s}_Z^2 = 0.23122$, is also indicated.) The ν_e-e [132,133] and $\bar{\nu}_e$-e [134] constraints are at 1 σ, while each of the four equivalent $\nu_\mu(\bar{\nu}_\mu)$-e [129–131] solutions ($g_{V,A} \to -g_{V,A}$ and $g_{V,A} \to g_{A,V}$) are at the 90% CL. The global best fit region (shaded) almost exactly coincides with the corresponding $\nu_\mu(\bar{\nu}_\mu)$-e region. The solution near $g_A = 0$ and $g_V = -0.5$ is eliminated by $e^+e^- \to \ell^+\ell^-$ data under the weak additional assumption that the neutral current is dominated by the exchange of a single Z boson.

e^+e^- annihilation

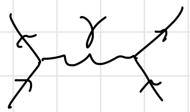
Now we turn to studying what we can learn from e^+e^- annihilations. In particular, we will start by studying $e^+e^- \to \mu^+\mu^-$.

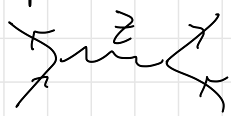
This will have the same amplitude as $e^+e^- \to \tau^+\tau^-$, but not $e^+e^- \to e^+e^-$ (why?).

Want to probe the theory from $E \ll m_Z$ to $E \sim m_Z$

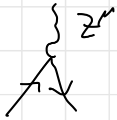
Fermi Theory (EFT) breaks down and we need UV Theory.

$\Rightarrow$ The tree-level amplitude is computed from

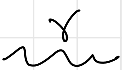
$i\mathcal{M} = i\mathcal{M}_\gamma + i\mathcal{M}_Z =$ 

$+$ 

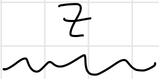
w/ 

$= -ie\gamma^\mu$; 

$= i\frac{g}{4c_W}(g_V - g_A\gamma^5)\gamma^\mu$



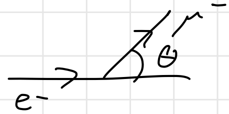
$= -\frac{i}{p^2 + i\varepsilon}\eta^{\mu\nu}$



$= -\frac{i}{p^2 - m_Z^2 + im_Z\Gamma_Z}\left(\eta^{\mu\nu} - \frac{p_\mu p_\nu}{m_Z^2}\right)$

is the Breit-Wigner propagator (see H/W)

and Γ_Z is the total width of the Z-boson.

We want to compute $\frac{d\sigma}{d\cos\theta}$ w/ 

in the limit with $m_e, m_\mu \to 0$.

$$\Rightarrow \frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{2s}\left[A(1+\cos^2\theta) + B\cos\theta\right] \qquad \left(\alpha = \frac{e^2}{4\pi}\right)$$

$$\text{w/ } A = \underbrace{1}_{\gamma} - \underbrace{\frac{8\sqrt{2}}{e^2} g_V^2 s \,\mathrm{Re}\left[G_F \bar{K}(s)\right]}_{\gamma\text{-}Z \text{ interference}} + \underbrace{\frac{32}{e^4}(g_V^2+g_A^2)^2 s^2 |G_F \bar{K}(s)|^2}_{Z}$$

$$B = -\underbrace{\frac{16\sqrt{2}}{e^2} g_A^2 s \,\mathrm{Re}\left[G_F \bar{K}(s)\right]}_{\gamma\text{-}Z \text{ interference}} + \underbrace{\left(\frac{16}{e^2}\right)^2 g_V^2 g_A^2 s^2 |G_F \bar{K}(s)|^2}_{Z}$$

and $\quad \bar{K}(s) \equiv \dfrac{m_Z^2}{m_Z^2 - s + i m_Z \Gamma_Z}$

Clearly in QED alone, would have $A=1$ & $B=0$.

Recall that QED preserves parity while the weak interactions do not. Suppose we had a beam of polarized electrons and positrons, so that

$$\overset{\hat{z}\;\rightarrow}{}\qquad \underset{\Leftarrow}{e_L^- \longrightarrow} \quad \underset{\Leftarrow}{\longleftarrow e_R^+} \qquad\qquad \begin{array}{l}\longrightarrow \text{ momentum}\\ \Rightarrow \text{ helicity}\end{array}$$

Angular momentum conservation implies 

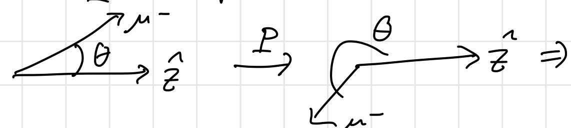
If we produce $\mu_L^- \mu_R^+$, then the configuration

$$\hat{z} \rightarrow \qquad \underset{\Leftarrow}{\mu_L^- \longrightarrow} \;\; \underset{\Leftarrow}{\longleftarrow \mu_R^+} \neq 0 \quad \text{while} \quad \hat{z}\rightarrow \quad \underset{\Leftarrow}{\mu_R^+ \longrightarrow} \;\; \underset{\Rightarrow}{\longleftarrow \mu_L^-} = 0$$

Indeed by direct calculation, we find (see 5.2 of Peskin + Schroeder)

$$|\mathcal{M}_{QED}(e_R^- e_L^+ \to \mu_R^- \mu_L^+)|^2 = |\mathcal{M}_{QED}(e_L^- e_R^+ \to \mu_L^- \mu_R^+)|^2$$
$$= e^4 (1+\cos\theta)^2$$

Note that P flips the direction of momentum,

$\Rightarrow$  $\Rightarrow$ $\theta \xrightarrow{P} \pi - \theta$, $\cos\theta \xrightarrow{P} -\cos\theta$

but it does not change the spin $\Rightarrow$ flips the helicity. So we can infer (also check by direct calculation) that

$$|\mathcal{M}_{QED}(e_R^- e_L^+ \to \mu_L^- \mu_R^+)|^2 = |\mathcal{M}_{QED}(e_L^- e_R^+ \to \mu_R^- \mu_L^+)|^2$$
$$= e^4 (1-\cos\theta)^2$$

This is consistent with angular momentum argument.

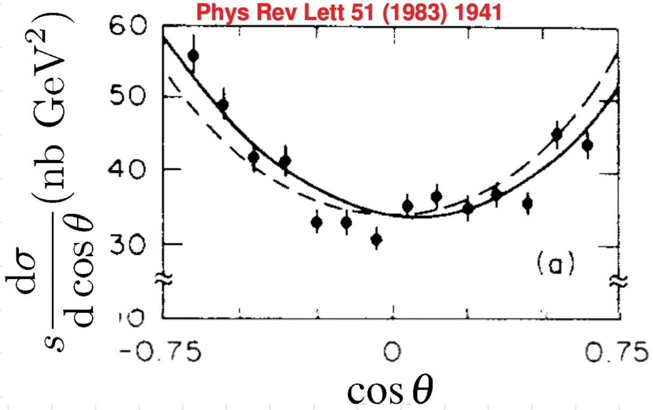
Finally, note $e_L^- e_L^+$ and $e_R^- e_R^+$ do not contribute in QED (w/ massless fermions) also by conservation of angular momentum. $\Rightarrow$ Average over initial state helicity (unpolarized beam)

$$\Rightarrow (1+\cos\theta)^2 + (1-\cos\theta)^2 \Rightarrow \frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{2s}(1+\cos^2\theta)$$

This also shows that $B \neq 0$ corresponds to violating parity.

$e^+e^- \to \mu^+\mu^-$ @ $\sqrt{s} = 29$ GeV

Mark II Expt., M.E.Levi et al.,
Phys Rev Lett 51 (1983) 1941



- - - - - **pure QED, O(α^3)**

——— **QED plus Z contribution**

Angular distribution becomes slightly asymmetric in higher order QED or when Z contribution is included

e.g. finite mass effects

Can define an observable that allows us to maximize statistical power of the data to be sensitive to $B \neq 0$: the forward-backward asymmetry.

$$\sigma_F = \int_0^1 d\cos\theta \, \frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{2s}\left[\frac{4}{3}A + \frac{1}{2}B\right]$$

$$\sigma_B = \int_{-1}^0 d\cos\theta \, \frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{2s}\left[\frac{4}{3}A - \frac{1}{2}B\right]$$

and the total cross section $\sigma_{tot} = \sigma_F + \sigma_B$

$$\Rightarrow \quad \sigma_{TOT} = \frac{4\pi^2\alpha^2}{3s} A$$

$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} = \frac{3B}{8A}$$

We can now study the phenomenology of $e^+e^- \to \mu^+\mu^-$ in two energy regimes $E \ll m_Z$ and $E \sim m_Z$.

Note that $G_F \overline{K}(s) \simeq G_F \Rightarrow$ Fermi theory.

$\Rightarrow$ Leading contribution from the weak interactions is due to the interference w/ QED

$$\Rightarrow A = 1 - \frac{8\sqrt{2}}{e^2} g_V^2 G_F s$$

Note $A \sim g_V^2$ while

$$B = -\frac{16\sqrt{2}}{e^2} g_A^2 G_F s$$

$B \sim g_A^2$ in this limit

$$\Rightarrow \sigma_{Tot} \simeq \frac{4\pi\alpha^2}{3s}\left[1 - \frac{8\sqrt{2}}{e^2} g_V^2 G_F s\right]$$

and $$\Delta_{FB} \simeq \frac{3}{8} B = -\frac{6\sqrt{2}}{e^2} g_A^2 G_F s$$

This allowed us to resolve the 2-fold ambiguity from the $e\nu$ scattering experiments to provide the first measurement of s_W^2.

$\Rightarrow$ We knew $s_W^2 \simeq 0.23$, $v = 246$ GeV (from μ-decay), and $e \simeq 0.30$ from Hydrogen

$\Rightarrow$ Could predict g and g'

$\Rightarrow$ Could predict $m_W = \frac{1}{2} g v \simeq 80$ GeV, $m_Z \simeq \frac{m_W}{c_W} \simeq 91$ GeV

After the W + Z were observed directly, the "Large Electron-Positron" (LEP) collider was built to study the detailed properties of the Z, by colliding e^+e^- at $E \simeq m_Z$.

For E near m_Z, the Z boson is "on-resonance" and so it completely dominates the physics.

$$\mathcal{K}(m_Z^2) \simeq \frac{m_Z}{i\Gamma_Z} \sim \frac{1}{i g^2/4\pi} \gg 1$$

Also note $Re[\mathcal{K}(m_Z^2)] \simeq 0$

$\Rightarrow$ γ-Z interference vanishes and pure Z-term dominates

Note $\Gamma_Z \simeq 2.5$ GeV. Let's vary the energy about m_Z by an amount set by δ:

$$s = m_Z^2(1+\delta) \quad w/ \quad \delta \sim \frac{\Gamma_Z}{m_Z} = 0.027$$

$$\Rightarrow A \simeq \frac{g^4}{e^4 c_W^4}(g_V^2 + g_A^2)\frac{1}{\delta^2 + \Gamma_Z^2/m_Z^2}$$

$$B \simeq \frac{8g^4}{e^4 c_W^4} g_V^2 g_A^2 \frac{1}{\delta^2 + \Gamma_Z^2/m_Z^2}$$

$$\Rightarrow \sigma_{Tot} \simeq \frac{g^4}{12\pi c_W^4 m_Z^2}(g_V^2 + g_A^2)^2 \frac{1}{\delta^2 + \Gamma_Z^2/m_Z^2}$$

We can measure the peak of σ_{Tot} @ the "Z pole" $s = m_Z^2$ $(\delta = 0)$

$$\Rightarrow \sigma_{peak} = \sigma_{Tot}(m_Z^2) = \frac{g^4}{12\pi c_W^4} (g_V^2 + g_A^2)^2 \frac{1}{\Gamma_Z^2}$$

We can also vary the energy to $s = m_Z^2 \pm m_Z \Gamma_Z$

$$\Rightarrow \sigma_{tot}(m_Z^2 \pm m_Z \Gamma_Z) = \frac{1}{2} \sigma_{peak}$$

Taken together, we get very precise measurement of m_Z (by position of peak), Γ_Z, and s_W^2 using $g_V = -\frac{1}{4}(1 - 4 s_W^2)$ & $g_A = -\frac{1}{4}$

$$\Rightarrow m_Z = 91.1876(21) \text{ GeV}$$

$$\Gamma_Z = 2.4952(23) \text{ GeV}$$

$$s_W^2 = 0.23116(12)$$

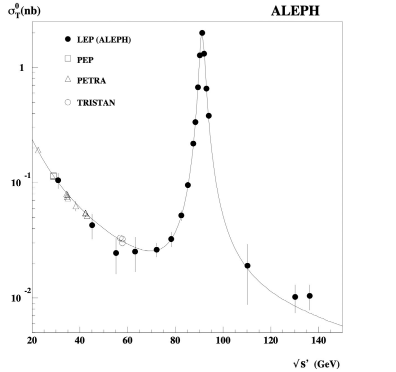


Figure 9: e^+e^- annihilation cross-section as a function of the collision energy, for the specific channel $e^+e^- \rightarrow \mu^+\mu^-$.

From Fernandez "Physics at LEP1 & LEP2" (2000)

The Narrow Width Approximation

In the narrow width limit $\Gamma \ll m$, we can treat an intermeadiate state as being like a "real" or "on-shell" particle. In this limit, the amplitude to produce this particle factorizes from the amplitude for it to decay. We will see that this approximation is extreamly useful in the following.

Note that

$$\int_{-\infty}^{\infty} ds \left| \frac{i}{s-m^2+im\Gamma} \right|^2 = \int_{-\infty}^{\infty} ds \frac{1}{(s-m^2)^2+m^2\Gamma^2} = \frac{\pi}{m\Gamma}$$

$$\Rightarrow \lim_{\Gamma \to 0} \int_{m^2-\Delta m^2}^{m^2+\Delta m^2} ds \frac{m\Gamma/\pi}{(s-m^2)^2+m^2\Gamma^2} f(s) = f(m^2)$$

or $$\left. \frac{1}{(s-m^2)^2+m^2\Gamma^2} \right|_{s \simeq m^2} \xrightarrow{\Gamma \ll m^2} \frac{\pi}{m\Gamma} \delta(s-m^2)$$

Then if we are interested in the Z-boson decay to many final states $X = \mu^+\mu^-, \tau^+\tau^-, q\bar{q}, \ldots$

Then we can define the "partial width" $\Gamma(Z \to X)$ for each final state so that $\Gamma_{tot} = \sum_X \Gamma(Z \to X)$. Then it is useful to define "branching ratios"

$$BR(Z \to X) = \frac{\Gamma(Z \to X)}{\Gamma_{tot}} \quad \text{s.t.} \quad 0 \leq BR(Z \to X) \leq 1$$

Then we can approximate $\sigma_X(s = m_Z^2)$ by

$$\sigma_{e^+e^- \to Z}(s = m_Z^2) \times BR(Z \to X) \quad \text{for } \Gamma_Z \ll m_Z$$

This is extreamly useful if we can differentiate the final states, since then we can take ratios and determine the BRs.

For example, one can show that the rate for $Z \to$ hadrons (the strongly coupled asymptic states of quarks and gluons) is given by

$$\sigma_{e^+e^- \to \text{hadrons}}(s = m_Z^2) \simeq \frac{12\pi}{m_Z^2 \Gamma_Z^2} \Gamma(Z \to e^+e^-)\, \Gamma(Z \to \text{hadrons})$$

$e^+e^- \to$ hadrons and the number of ν's.

A very naive model of e^+e^- to hadrons is to just compute $e^+e^- \to q\bar{q}$

$$\Rightarrow \quad \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = \sum_f 3 Q_f^2 = \begin{cases} 2 & u, d, s \\ 10/3 & u, d, s, c \\ 11/3 & u, d, s, c, b \end{cases}$$

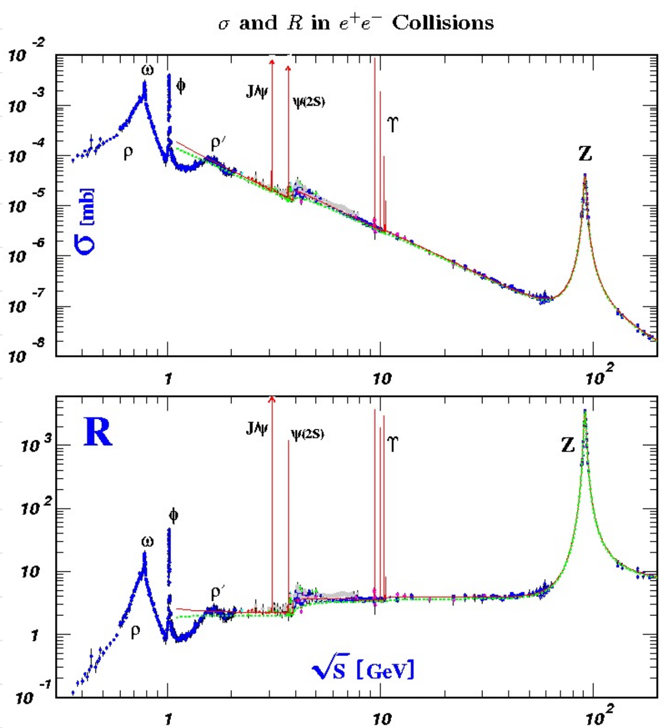
(# colors)

Can also include leading loop correction (including running coupling)

$$\frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = \sum_f 3 Q_f^2 \left(1 + \frac{\alpha_s(s)}{\pi} + \dots\right)$$

green: no loop correction

red: with loop corrections



We can learn one more important lesson from the Z-line shape. It can give us an indirect measurement of the number of neutrinos. To understand this, first note that we need to include the effect of initial state radiation (ISR). There is an IR divergence due to the ability to emit soft and collinear photons. The leading effect is to decrease the total rate by a factor of a "Sudakov double log" and to push the resonance to slightly higher energies.

This is a very interesting topic that we may discuss in more detail during lecture if there is time.

To extract the number of neutrinos, we use the fact that the total Z width is given by

$2.49 \text{ GeV} = \Gamma_Z$ 6.20

$$= \sum_{e,\mu,\tau} \Gamma(Z \to l^+ l^-) + \Gamma(Z \to \text{hadrons}) + N_\nu \Gamma(Z \to \nu\bar{\nu})$$

We can compute

$$\Gamma(Z \to l^+ l^-) = N_{lep} \times 83.4 \text{ MeV}$$

$$\Gamma(Z \to \nu\bar{\nu}) = N_\nu \times 166 \text{ MeV}$$

and we will extract $\Gamma(Z \to \text{hadrons})$ from data.

By measuring all processes that generate hadrons at $s = m_Z^2$ and using the narrow width approx.

$$\Rightarrow \Gamma(Z \to \text{hadrons}) \simeq 1.734(15) \text{ GeV}$$

$$\Rightarrow N_\nu \simeq 2.9841(83)$$

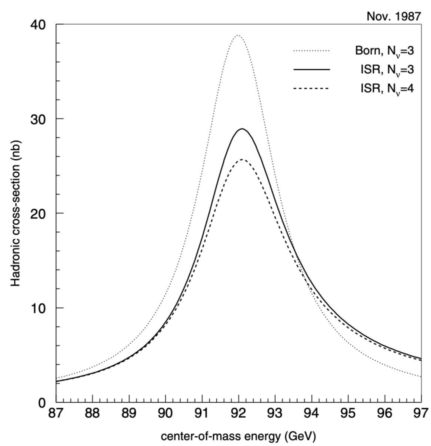


Fig. 1. The $e^+e^- \to hadrons$ cross section as a function of center-of-mass energy. This curve was drawn before LEP start-up in 1987. At that time the Z mass was measured to be around 92 GeV with an error larger than 1.5 GeV. The dotted line represents the born approximation prediction for three species of light neutrinos. The full line includes the effect of initial state radiation. The dashed line represents the effect of adding one more type of light neutrino with the same couplings as the first three. It is clear from this picture that the cross-section at the peak of the resonance contains most of the information on the number of light neutrino species.

Blondel from PDG

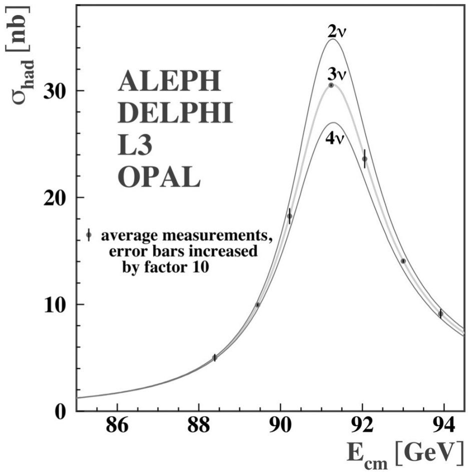


Fig. 4. The $e^+e^- \to$ hadrons cross section as a function of center-of-mass energy, as measured by the LEP experiments. The curves represent the Standard Theory predictions for two, three and four species of light neutrinos. It is clear from this picture that there is no further light neutrino species with couplings identical to the first three.

(Can also discuss precision electroweak if there is time...)